Midterm 1

This midterm has two questions worth 45 points each. Show all your work. Good luck!

Question 1

After a few periods of sit on their island's beach trading coconuts, Jack and Barbossa decide to move further inland. Jack discovers a palm that grows dates, whereas Barbossa finds an orange tree. Jack's date palm drops E_D dates every period, and Barbossa's orange tree produces E_O oranges. Both Jack and Barbossa have preferences over dates and oranges according to the period utility function $u(c_D, c_O) = \log c_D + \gamma \log c_O$. Both have a discount factor of $0 < \beta < 1$. Jack and Barbossa plan to stay in that area for 2 periods and aggree to trade dates and oranges.

1. Carefully define a competitive Arrow-Debreu equilibrium. (Note: this includes stating the maximization problem explicitly.) (10 points)
2. Solve for the equilibrium. (25 points)
3. Explain the results in economic terms and the intuition behind them. What determines the relative price of dates and oranges? What determines the relative price of oranges today vs oranges tomorrow? What is Jack and Barbossa's relative consumption of fruit? (10 points)

Question 2

A firm faces an inverse demand curve of $p = xy^{-\alpha}$ for its product (the inverse demand curve indicates the market price that the firm can sell each good for when it sells a total of y goods). It can produce goods with a linear production function (ie producing y goods costs wy, where w is the constant marginal cost of production). But in addition to the cost of production, it is costly to change the production process and there is a cost of *changing* production: If last period the firm produced y_{-1} goods, and it produces y goods this period, it has to pay an adjustment cost of $\frac{\chi}{2}\left(y - y_{-1}\right)^2$. So the total cost of production is $C\left(y, y_{-1}\right) = wy + \frac{\phi}{2}\left(y - y_{-1}\right)^2$.

The variable x in the firm's demand function denotes the firm's stock of customers: More customers means higher demand, which translates into a higher sales price. The firm increase its stock of customers by marketing: If it undertakes marketing expenses of an amount i today, then next period it will add $w\sqrt{i}$ new customers (note that the firm's total cost therefore consists of the cost of production plus the cost of marketing). However, it also loses customers over time. Specifically, it will lose a share δ of its current customers by next period.

1. Write down the firm's profits for a single period. Point out which variables the firm takes as given in this period, and which variables the firm can choose. (5 points)

2. Now write down the firm's value function if it wants to maximize profits over an infinite horizon and discounts future profits at a rate β. Make sure to include stating the constraint set. Clearly denote state variable(s) and control variable(s). (15 points)

3. Write down the first-order condition(s) associated with the firm maximization problem. Also find the envelope condition(s), and combine envelope condition(s) and FOC(s) to find the intertemporal optimality condition(s) analogous to the Euler equation in the household problem. (15 points)

4. Find the steady-state for the firm. How many customers will it have in the long run, and how many goods does it produce? (10 points)

Solutions Question 1

1. An ADE is defined as an allocation $\left\{\left(c_{Ot}^J, c_{Dt}^J\right), \left(c_{Ot}^B, c_{Dt}^B\right)\right\}_{t=0,1}$ and prices $\{(p_{Ot}, p_{Dt})\}_{t=0,1}$ such that

- given prices, the allocation solves Jack and Barbossa's problem, ie for $i = J, B$

$$\max_{c_{O,0}^i, c_{D0}^i, c_{O1}^i, c_{D1}^i} \sum_{t=0}^{1} \beta^t \log c_D + \gamma \log c_O$$
$$\text{s.t.} \quad \sum_{t=0}^{1} p_{Ot} c_{Ot}^i + p_{Dt} c_{Dt}^i = \sum_{t=0}^{1} p_{Ot} e_{Ot}^i + p_{Dt} e_{Dt}^i$$

- markets clear, ie for $t = 0, 1$

$$\begin{aligned} c_{Ot}^J + c_{Ot}^B &= E_O \\ c_{Dt}^J + c_{Dt}^B &= E_D \end{aligned}$$

1. From the FOCs, for $t = 0, 1$

$$\begin{aligned} \beta^t \frac{1}{c_{Dt}^i} &= \lambda p_{Dt} \\ \beta^t \frac{\gamma}{c_{Ot}^i} &= \lambda p_{Ot} \end{aligned}$$

so that

$$\begin{aligned} \frac{c_{D,t+1}^i}{c_{Dt}^i} &= \beta \frac{p_{Dt}}{p_{D,t+1}} \\ c_{Dt}^i &= \frac{1}{\beta} \frac{p_{D,t+1}}{p_{Dt}} c_{D,t+1}^i \end{aligned}$$

From this we can use market clearing to write

$$\begin{aligned} c_{Dt}^J + c_{Dt}^B &= \frac{1}{\beta} \frac{p_{D,t+1}}{p_{Dt}} c_{D,t+1}^J + \frac{1}{\beta} \frac{p_{D,t+1}}{p_{Dt}} c_{D,t+1}^B \\ \beta \frac{E_{Dt}}{E_{D,t+1}} &= \frac{p_{D,t+1}}{p_{Dt}}. \end{aligned}$$

We also know from the FOCs that

$$\begin{aligned} \frac{c_{Ot}^i}{c_{Dt}^i} &= \gamma \frac{p_{Dt}}{p_{Ot}} \\ c_{Ot}^i &= \gamma \frac{p_{Dt}}{p_{Ot}} c_{Dt}^i \end{aligned}$$

so that again

$$\begin{aligned}
c^J_{Ot} + c^B_{Ot} &= \gamma_J \frac{p_{Dt}}{p_{Ot}} c^J_{Dt} + \gamma_B \frac{p_{Dt}}{p_{Ot}} c^B_{Dt} \\
& \frac{p_{Dt}}{p_{Ot}} \left(\gamma_J c^J_{Dt} + \gamma_B c^B_{Dt} \right) \\
\frac{\gamma E_{Dt}}{E_{Ot}} &= \frac{p_{Ot}}{p_{Dt}}.
\end{aligned}$$

Since we can normalize one price, let's choose $p_{D0} = 1$. We can use the budget constraint to find

$$\begin{aligned}
p_{O,0} c^J_{O,0} + p_{D0} c^J_{D0} + p_{O1} c^J_{O1} + p_{D1} c^J_{D1} &= p_{O,0} E_O + p_{O1} E_O \\
p_{O,0} \gamma \frac{p_{D0}}{p_{O,0}} c^i_{D0} + p_{D0} c^J_{D0} + p_{O1} \gamma \frac{p_{D1}}{p_{O1}} c^J_{D1} + p_{D1} c^J_{D1} &= \frac{\gamma E_D}{E_{Ot}} E_O + p_{D1} \frac{\gamma E_{Dt}}{E_{Ot}} E_O \\
\gamma p_{D0} c^i_{D0} + p_{D0} c^J_{D0} + \gamma p_{D1} c^J_{D1} + p_{D1} c^J_{D1} &= \gamma E_D + \beta \gamma E_D \\
\gamma p_{D0} c^i_{D0} + p_{D0} c^J_{D0} + \gamma \beta p_{D0} c^J_{D0} + \beta p_{D0} c^J_{D0} &= \gamma E_D + \beta \gamma E_D
\end{aligned}$$