Final exam

Question 1 (55 points)

Soren the Swede lives far North in Sweden. His period utility function over consumption c and leisure x is given by $u(c_t, x_t)$. Soren's total time endowment is equal to 1 which he can split up into leisure or work l (you can ignore corner solutions for the labor-leisure choice). He works in the forest and can produce w_t goods per unit of labor. Every period one of N possible events can happen, so the event space is $S = \{1, \ldots, N\}$. Soren can buy a risk-free bond b_{t+1} at a price of q_t.

1. Everyone in Soren's Swedish village has preferences and work opportunities identical to Soren's, and the villagers trade amongst each other. Define a competitive equilibrium. (10 points)

Now assume $N = 2$, and that Soren's utility value of leisure varies stochastically: enjoying leisure when the sun shines gives him more utility than having leisure when it rains. His preferences now are represented by the utility function

$$u(c_t, x_t) = (c_t^\sigma + \phi_t x_t^\sigma)^{\frac{1}{\sigma}},$$

where $\phi_t \in \{\phi^h, \phi^l\}$ is a random variable with probabilities π^h and $\pi^l = 1 - \pi^h$ as well as $\phi^h > \phi^l$. In addition assume that Soren will retire in period 2, ie he only considers his utility in periods 0 and 1. Soren maximizes expected utility.

2. In equilibrium, how does labor in period 1 depend on ϕ_t and w? Is it true that if leisure is valued highly ($\phi_t = \phi^h$) Soren will spend less time at work? Is it true that if the wage is high Soren will spend more time at work? Explain the intuition of your result. (15 points)

3. Show that if leisure and consumption are substitutes ($0 < \sigma < 1$), then the marginal utility of consumption is increasing in ϕ_t, and if they are complements ($\sigma < 0$) it is decreasing in ϕ_t. (15 points)

4. Let's say consumption and leisure are complements. Now assume there is a generic asset a_1 available which can be bought at time 0 for a price of p_0^a. In period 1 it will pay off $d_1 \in \{\hat{d}, \tilde{d}\}$, depending on the state of the world: It pays off $\hat{d}$ if $\phi_1 = \phi^h$ and $\tilde{d}$ if $\phi_1 = \phi^l$. From the first-order conditions, price the asset in terms of stochastic discount factor and asset returns.

   - Is the price of the asset higher if today's state is ϕ^h of if it is ϕ^l? Explain intuitively.
   - Is the price of the asset higher if $\hat{d} > \tilde{d}$ or if $\hat{d} < \tilde{d}$? Explain intuitively
   - Can we tell whether the price of the asset is increasing or decreasing in π^h? Explain intuitively.

   (You may find the formula for the expectation of products $\text{Cov}(X, Y) = E[X, Y] - E[X]E[Y]$ useful.) (15 points)

Question 2 (35 points)

Wanda the worker currently holds a job that pays a wage w_t and she works full time (i.e. supplies one unit of labor). She can decide how much effort e_t to put into her work. Spending more effort will improve the quality of her work – while this has no effect on her current income, it will advance her career and cause her wage to raise in the future. Specifically, the raise she gets between this period and the next depends on the effort spent e_t through $r(e_t) = e_t - \theta$. For example, if $r(e_t) = 0$, then next period her wage will be the same as it is today. She doesn't borrow or save, discounts the future at a rate of β, and consumes her wage payments directly from which she derives utility of $\ln w_t$, but expending effort causes her disutility of $\frac{\nu}{2}e_t^2$ (where ν is a constant).

1. Set up Wanda's value function, and clearly denote the respective state and control variables. (7 points)

2. From the value function derive the intertemporal optimality condition. (7 points)

3. Which wage level will Wanda reach eventually? Show whether and how it depends on the parameters θ,ν and β. (7 points)

The Dynamic Programming approach easily extends to the stochastic case. Now assume that, instead of getting a deterministic raise as a function of the effort level, Wanda will stochastically either get a raise so that $w_{t+1} = \gamma w_t$ (where $\gamma > 1$), or will get no raise so that $w_{t+1} = w_t$. While Wanda can no longer affect the size of the raise, putting in more effort now affects the probability of getting a raise. This probability of getting the raise is given by the increasing and concave function $p(e)$ and of course this implies that the probability of staying at the old wage level is given by $1 - p(e)$.

4. State Wanda's new value function, keeping in mind that the continuation value is now an expected value. (7 points)

5. What is Wanda's new optimality condition? (7 points)